XXI. " On the Thermal Effects of Fluids in Motion :—Tempe-
rature of a Body moving slowly through Air." By Prof.
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The motion of air in the neighbourhood of a body moving very
slowly through it, may be approximately determined by treating the
problem as if air were an incompressible fluid. The ordinary hydro-
dynamical equations, so applied, give the velocity and the pressure
of the fluid at any point ; and the variations of density and tempe-
rature actually experienced by the air are approximately determined
by using the approximate evaluation of the pressure thus obtained.
Now, if a solid of any shape be carried uniformly through a perfect
liquid *, it experiences fluid-pressure at different parts of its surface,
expressed by the following formula,—

$$p = \Pi + \tfrac{1}{2}\rho \ (V^2 - q^2),$$

where Π denotes the fluid-pressure at considerable distances from the
solid, ρ the mass of unity of volume of the fluid, V the velocity of
translation of the solid, and q the velocity of the fluid relatively to
the solid, at the point of its surface in question. The effect of this
pressure on the whole is, no resultant force, and only a resultant
couple which vanishes in certain cases, including all in which the
solid is symmetrical with reference to the direction of motion. If
the surface of the body be everywhere convex, there will be an aug-
mentation of pressure in the fore and after parts of it, and a dimi-
nution of pressure round a medium zone. There are clearly in every
such case just two points of the surface of the solid, one in the fore
part, and the other in the after part, at which the velocity of the fluid
relatively to it is zero, and which we may call the fore and after pole
respectively. The middle region round the body in which the relative
velocity exceeds V, and where consequently the fluid pressure is dimi-
nished by the motion, may be called the equatorial zone ; and where
there is a definite middle line, or line of maximum relative velocity,
this line will be called the equator.

* That is, as we shall call it for brevity, an ideal fluid, perfectly incompressible
and perfectly free from mutual friction among its parts.

If the fluid be air instead of the ideal "perfect liquid," and if the motion be slow enough to admit of the approximation referred to above, there will be a heating effect on the fore and after parts of the body, and a cooling effect on the equatorial zone. If the dimensions and the thermal conductivity of the body be such that there is no sensible loss on these heating and cooling effects by conduction, the temperature maintained at any point of the surface by the air flowing against it, will be given by the equation

$$t = \Theta\left(\frac{p}{\Pi}\right)^{\frac{\cdot 41}{1\cdot 41}},$$

where Θ denotes the temperature of the air as uninfluenced by the motion, and p and Π denote the same as before*. Hence, using for p its value by the preceding equation, we have

$$t = \Theta\left\{1 + \frac{\rho}{2\Pi}\left(V^2 - q^2\right)\right\}^{\frac{\cdot 41}{1\cdot 41}}.$$

But if H denote the length of a column of homogeneous atmosphere, of which the weight is equal to the pressure on its perpendicular section, and if g denote the dynamical measure of the force of gravity (32·2 in feet per second of velocity generated per second), we have

$$g\rho\mathrm{H} = \Pi;$$

and if we denote by α the velocity of sound in the air, which is equal to $\sqrt{1\cdot 41 \times g\mathrm{H}}$, the expression for the temperature becomes

$$t = \Theta\left\{1 + \frac{1\cdot 41}{2} \cdot \frac{V^2 - q^2}{\alpha^2}\right\}^{\frac{\cdot 41}{1\cdot 41}}.$$

According to the supposition on which our approximation depends, that the velocity of the motion is small, that is, as we now see, a small fraction of the velocity of sound, this expression becomes

$$t = \Theta\left\{1 + \cdot 41 \times \frac{V^2 - q^2}{2\alpha^2}\right\}.$$

At either the fore or after pole, or generally at every point where the velocity of the air relatively to the solid vanishes (at a re-entrant

* The temperatures are reckoned according to the absolute thermodynamic scale which we have proposed, and may, to a degree of accuracy correspondent with that of the ordinary "gaseous laws," be taken as temperature Centigrade by the air-thermometer, with 273°·7 added in each case. See the author's previous paper "On the Thermal Effects of Fluids in Motion," Part II., Philosophical Transactions, 1854, part 2. p. 353.

angle for instanee, if there is such), we have $q = o$, and therefore an elevation of temperature amounting to

$$\cdot 41 \times \frac{V^2}{2\alpha^2} \Theta.$$

If, for instance, the absolute temperature, Θ, of the air at a distance from the solid be $287°$ (that is $55°$ on the Fahr. scale), for which the velocity of sound is 1115 per second, the elevation of temperature at a pole, or at any point of no relative motion, will be, in degrees Centigrade,

$$58°\cdot 8 \times \left(\frac{V}{\alpha}\right)^2, \text{ or } 58°\cdot 8 \times \left(\frac{V}{1115}\right)^2,$$

the velocity V being reckoned in feet per second. If, for instance, the velocity of the body through the air be 88 feet per second (60 miles an hour), the elevation of temperature at the points of no relative motion is $\cdot 36°$, or rather more than $\frac{1}{3}$ of a degree Centigrade.

To find the greatest depression of temperature in any case, it is necessary to take the form of the body into account. If this be spherical, the absolute velocity of the fluid backwards across the equator will be half the velocity of the ball forwards; or the relative velocity (q) of the fluid across the equator will be $\frac{3}{2}$ of the velocity of the solid. Hence the depression of temperature at the equator of a sphere moving slowly through the air will be just $\frac{9}{4}$ of the elevation of temperature at each pole. It is obvious from this that a spheroid of revolution, moving in the direction of its axis, would experience at its equator a depression of temperature, greater if it be an oblate spheroid, or less if it be a prolate spheroid, than $\frac{9}{4}$ of the elevation of temperature at each pole.

It must be borne in mind, that, besides the limitation to velocities of the body small in comparison with the velocity of sound, these conclusions involve the supposition that the relative motions of the different parts of the air are unresisted by mutual friction, a supposition which is not even approximately true in most cases that can come under observation. Even in the case of a ball pendulum vibrating in air, Professor Stokes[*] finds that the motion is seriously influenced

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[*] "On the Effect of the Internal Friction of Fluids on the Motion of Pendulums," read to the Cambridge Philosophical Society, Dec. 9, 1850, and published in vol. ix. part 2 of their Transactions.

by fluid friction. Hence with velocities which could give any effect sensible on even the most delicate of the ether thermometers yet made (330 divisions to a degree), it is not to be expected that anything like a complete verification or even illustration of the preceding theory, involving the assumption of no friction, can be had. It is probable that the forward polar region of heating effect will, in consequence of fluid friction, become gradually larger as the velocity is increased, until it spreads over the whole equatorial region, and does away with all cooling effects.

Our experimental inquiry has hitherto been chiefly directed to ascertain the law of the thermal effect upon a thermometer rapidly whirled in the air. We have also made some experiments on the modifying effects of resisting envelopes, and on the temperatures at different parts of the surface of a whirled globe. The whirling apparatus consisted of a wheel worked by hand, communicating rapid rotation to an axle, at the extremity of which an arm carrying the thermometer with its bulb outwards was fixed. The distance between the centre of the axle and the thermometer bulb was in all the experiments 39 inches. The thermometers made use of were filled with ether or chloroform, and had, the smaller 275, and the larger 330 divisions to the degree Centigrade. The lengths of the cylindrical bulbs were $\frac{9}{10}$ and $1\frac{4}{10}$ inch, their diameters ·26 and ·48 of an inch respectively.

<div align="center">TABLE I.—Small bulb Thermometer.</div>

Velocity in feet per second.	Rise of temperature in divisions of the scale.	Rise divided by square of velocity.
46·9	27½	·0125
51·5	32	·0121
68·1	46½	·0100
72·7	57½	·0109
78·7	67½	·0109
84·8	74	·0103
104·5	91	·0083
130·2	151	·0089
133·2	172	·0097
145·4	191	·0090

Mean.. ·01026

The above Table shows an increase of temperature nearly proportional to the square of the velocity.

$$V = \sqrt{\frac{275}{\cdot01026}} = 163 \cdot 7 = \text{the velocity in feet per second, which,}$$

in air of the same density, would have raised the temperature 1° Centigrade.

TABLE II.—Larger bulb Thermometer.

Velocity in feet per second.	Rise of temperature in divisions of scale.	Rise divided by square of velocity.
36·3	18	·0125
66·6	42	·0095
84·8	57	·0079
125·6	146	·0093
	Mean..	·0098

In this instance $V = \sqrt{\dfrac{330}{\cdot0098}} = 183 \cdot 5$ feet per second for 1° Centigrade. It is however possible that the full thermal effect was not so completely attained in three minutes (the time occupied by each whirling) as with the smaller bulb. On the whole it did not appear to us that the experiments justified the conclusion, that an increase of the dimensions of the bulb was accompanied by an alteration of the thermal effect.

TABLE III.—Larger bulb Thermometer covered with five folds of writing-paper.

Velocity in feet per second.	Rise of temperature in divisions of scale.	Rise divided by square of velocity.
36·3	20	·0152
51·5	43	·0162
72·6	53	·0101
118	132	·0095

The increased thermal effect at comparatively slow velocities, exhibited in the above Table, appeared to be owing to the friction of the air against the paper surface being greater than against the polished glass surface.

One quarter of the enveloping paper was now removed, and the bulb whirled with its bared part in the rear. The results were as follow:—

TABLE IV.—Paper removed from posterior side.

Velocity in feet per second.	Rise of temperature in divisions of scale.	Rise divided by square of velocity.
75·6	60	·0105
96·8	87	·0093

On whirling in the contrary direction, so that the naked part of the bulb went first, we got,—

TABLE V.—Paper removed from anterior side.

Velocity in feet per second.	Rise of temperature in divisions of scale.	Rise divided by square of velocity.
81·7	56	·0084
93·8	72	·0082

On rotating with the bare part, posterior and anterior in turns, at the constant velocity of 90 feet per second, the mean result did not appear to indicate any decided difference of thermal effect.

Another quarter of paper was now removed from the opposite side. Then on whirling so that the bared parts were anterior and posterior, we obtained a rise of 83 divisions with a velocity of 93·8. But on turning the thermometer on its axis one quarter round, so that the bared parts were on each side, we found the somewhat smaller rise of 62 divisions for a velocity of 90·8 feet per second.

The effect of surface friction having been exhibited at slow velocities with the papered bulb, we were induced to try the effect of increasing it by wrapping iron wire round the bulb.

TABLE VI.—Larger bulb Thermometer wrapped with iron wire.

Velocity in feet per second.	Rise in divisions of scale.	Rise divided by square of velocity.
15·36	10·25	·0434
23·04	33	·0623
30·71*	49·25	·0522
46·08	68·75	·0324
69·12	98	·0206
111·34	185	·0149
126·72	207	·0129
153·55	above 280	above ·0118

* The whirring sound began at this velocity. According to its intensity the

On inspecting the above Table, it will be seen that the thermal effect produced at slow velocities was five times as great as with the bare bulb. This increase is evidently due to friction. In fact, as one layer of wire was employed, and the coils were not so close as to prevent the access of air between them, the surface must have been about four times as great as that of the uncovered bulb. At high velocities, it is probable that a cushion of air which has not time to escape past resisting obstacles makes the actual friction almost independent of variations of surface, which leave the magnitude of the body unaltered. In conformity with this observation, it will be seen that at high velocities the thermal effect was nearly reduced to the quantity observed with the uncovered bulb. Similar remarks apply to the following results obtained after wrapping round the bulb a fine spiral of thin brass wire.

TABLE VII.—Bulb wrapped with a spiral of fine brass wire.

Velocity in feet per second.	Rise in divisions of scale.	Rise divided by square of velocity.
7·68	2·5	·0424
15·36	13·5	·0572
23·04	36·5	·0687
30·71	48	·0509
46·08	64·5	·0304
76·8	103·5	·0175
115·18	224·5	·0169
148·78	264	·0119

The thermal effects on different sides of a sphere moving through air, have been investigated by us experimentally by whirling a thin glass globe of 3·58 inches diameter along with the smaller thermometer, the bulb of which was placed successively in three positions, viz. in front, at one side, and in the rear. In each situation it was placed as near the glass globe as possible without actually touching it.

thermal effect must necessarily suffer diminution ; unless indeed it gives rise to increased resistance.

TABLE VIII.—Smaller Thermometer whirled along with glass globe.

Velocity in feet per second.	Rise in divisions of scale.		
	Therm. in front.	Therm. at side.	Therm. in rear.
3·84	·66	10	4
7·68	2·66	40	10·5
15·36	41·9	78	51
23·04	71·2	90	71·7
38·4	78·4	90	68
57·5	99·9	112	76
70·92			107

The effects of fluid friction are strikingly evident in the above results, particularly at the slow velocities of 3 and 7 feet per second. It is clear from these, that the air, after coming in contact with the front of the globe, traverses with friction the equatorial parts, giving out an accumulating thermal effect, a part of which is carried round to the after pole. At higher velocities the effects of friction seem rapidly to diminish, so that at velocities between 23 and 38 feet per second, the mean indication of thermometers placed all round the globe would be nearly constant. Our anticipation (written before these latter experiments were made), that a complete verification of the theory propounded at the commencement was impossible with our present means, is thus completely justified.

It may be proper to observe, that in the form of experiment hitherto adopted by us, the results are probably, to a trifling extent, influenced by the vortex of air occasioned by the circular motion.

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We have on several occasions noticed the effect of sudden changes in the force of wind on the temperature of a thermometer held in it. Sometimes the thermometer was observed to rise, at other times to fall, when a gust came suddenly on. When a rise occurred, it was seldom equivalent to the effect, as ascertained by the foregoing experiments, due to the increased velocity of the air. Hence we draw the conclusion, that the actual temperature of a gust of wind is lower than that of the subsequent lull. This is probably owing to the air in the latter case having had its *vis viva* converted into heat by

collision with material objects. In fact we find that in sheltered situations, such for instance as one or two inches above a wall opposite to the wind, the thermometer indicates a higher temperature than it does when exposed to the blast. The question, which is one of great interest for meteorological science, has hitherto been only partially discussed by us, and for its complete solution will require a careful estimate of the temperature of the earth's surface, of the effects of radiation, &c., and also a knowledge of the causes of gusts in different winds.

XXII. "On the Thermal Effects of Longitudinal Compression of Solids." By J. P. Joule, Esq., F.R.S.; and "On the Alterations of Temperature accompanying Changes of Pressure in Fluids." By Prof. W. Thomson, F.R.S. Received June 18, 1857.

In the further prosecution of the experiments of which an outline was given in the 'Proceedings' for January 29, 1857, the author has verified the theory of Professor Thomson, as applied to the thermal effects of laying weights on and taking them off metallic pillars and cylinders of vulcanized india-rubber. Heat is evolved by compression, and absorbed on removing the compressing force in every substance yet experimented on. In the case of metals, the results agree very closely with the formula in which e, the longitudinal expansion by heat under pressure, is considered the same as the expansion without pressure. It was observed, however, that all the experimental results were a little in excess of the theoretical, and it became therefore important to inquire whether the force of elasticity in metals is impaired by heat. In the first arrangements for this purpose, the actual expansion of the bars employed in the experiments was ascertained by a micrometric apparatus,—1st, when there was no tensile force, and 2nd, when a weight of 700 lbs. was hung to the extremity of the quarter-inch rods. The results, reliable to less than one-hundredth of their whole value, did not exhibit any notable effect of tensile force on the coefficient of expansion by heat. An experiment susceptible of greater delicacy